MOMENT OF INERTIA

EXPERIMENT PROCEDURE

- **Determine the torsional coefficient D_r between for the springs used to couple the objects.**
- **Determine the moment of inertia J for a dumbbell bar without any added weights**
- **Determine the moment of inertia J as a function of distance r of a weight from its axis of rotation.**
- **Determine the moment of inertia J for a circular wooden disc, a wooden sphere and both solid and hollow cylinders**

OBJECTIVE

Determine the moment of inertia for various test bodies.

SUMMARY

A body's moment of inertia around an axis of rotation depends on how the mass of the object is distributed with respect to the axis. This will be investigated for a dumbbell, which has two weights symmetrically aligned either side of the axis, for a circular wooden disc, a wooden sphere and both solid and hollow cylinders. The period of oscillation of the test bodies is dependent on the mass distribution and the effective radius of the object.

REQUIRED APPARATUS

Quantity	Description	Number
1	Torsion Axle	**1008662**
1	Photo Gate	**1000563**
1	Digital Counter (230 V, 50/60 Hz)	**1001033 or**
	Digital Counter (115 V, 50/60 Hz)	**1001032**
1	Barrel Foot, 1000 g	**1002834**
1	Tripod Stand 185 mm	**1002836**
1	Precision Dynamometer 1 N	**1003104**
1	Set of Test Bodies for Torsion Axle	**1008663**

GENERAL PRINCIPLES

The inertia of a rigid body with respect to a change in its rotational motion around a fixed axis is given by its moment of inertia J. This is dependent on the distribution of mass in the body relative to the axis of rotation and increases at greater distance from the axis of rotation itself.

In general, moment of inertia is defined by means of a volume integral:

(1) $$J = \int_V r_s^2 \cdot \rho(r) \cdot dV$$

r_s: component of r perpendicular to the axis of rotation
$\rho(r)$: Distribution of mass in the body

Using as an example a dumbbell set, which has two weights of mass m symmetrically arranged at a distance r from the axis of rotation, then the moment of inertia is as follows:

(2) $$J = J_0 + 2 \cdot m \cdot r^2$$

J_0: Moment of inertia of dumbbell bar without weights

Now we can attach various test bodies to a twisting axis so that they can oscillate. If the period of oscillation is T, then the following is true:

(3) $$T = 2\pi \cdot \sqrt{\frac{J}{D_r}}$$

D_r: Torsional coefficient of coil springs

The means that the period of oscillation T will be greater when the moment of inertia J is larger.
The torsional coefficient of the coil springs can be determined with the help of a spring dynamometer:

(4) $$D_r = \frac{F \cdot r}{\alpha}$$

α: Deflection from equilibrium state

EVALUATION

From equation (3) it is possible to obtain a formula for determining the moment of inertia: $$J = D_r \cdot \frac{T^2}{4\pi^2}$$

For the set-up involving the dumbbell, it is then necessary to subtract the moment of inertia of the bar itself: J(weights) = J(bar + weights) - J(bar)

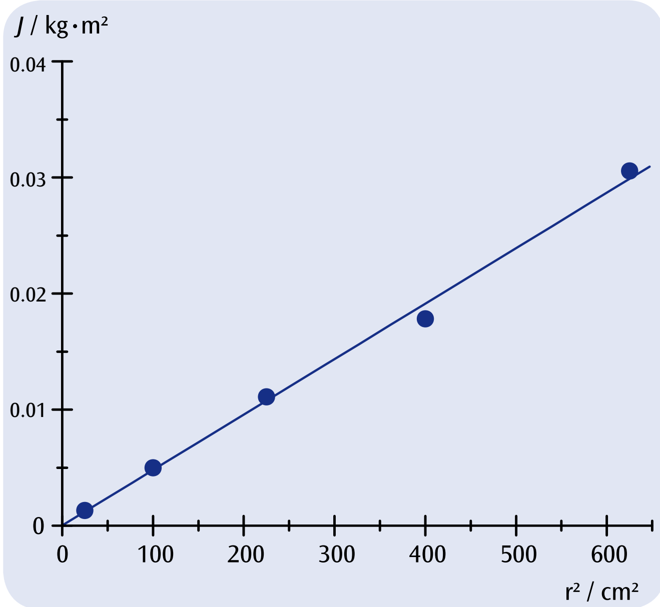

Fig. 1: Moment of inertia J of weights as a function of their radius r from the axis of rotation