PRECESSION AND NUTATION OF A GYROSCOPE

OBJECTIVE

Experimental investigation of precession and nutation of a gyroscope and determination of moment of inertia.

SUMMARY

A spinning disc exhibits motions known as precession and nutation in addition to its rotational motion, depending on whether there is an external force, and thereby an additional torque, acting upon its axle or if the axle of a disc spinning in an equilibrium state is then deflected from its equilibrium position. The period of precession is inversely proportional to the period of rotation while the period of nutation is directly proportional to the period of rotation. The way the period of precession depends on the period of rotation makes it possible to determine the moment of inertia of the rotating disc.

EXPERIMENT PROCEDURE

- **Verify that the frequency of rotation f_R of a rotating disc is proportional to the period of precession of a gyroscope T_P and determine the moment of inertia by plotting f_R (T_P).**
- **Verify that the frequency of rotation f_R of a rotating disc is proportional to the frequency of nutation f_N by plotting f_N (f_R) or the corresponding periods T_R (T_N).**

REQUIRED APPARATUS

Quantity	Description	Number
1	Gyroscope	1000695
2	Photo Gate	1000563
1	Laser Diode, Red	1003201
1	3B NET*log*™ (230 V, 50/60 Hz)	1000540 or
	3B NET*log*™ (115 V, 50/60 Hz)	1000539
1	3B NET*lab*™	1000544
3	Tripod Stand 150 mm	1002835
3	Universal Clamp	1002830
3	Stainless Steel Rod 750 mm	1002935

GENERAL PRINCIPLES

A spinning top is a rigid body which spins around an axis fixed at a given point. If an external force acts upon the axis, its torque causes a change in the angular momentum. The top then moves in a direction perpendicular to the axis and the force acting upon it. Such a motion is called precession. If a top is pushed away from its axis of rotation its starts to undergo a tipping motion. This motion is called nutation. In general, both these motions occur superimposed on one another.

In this experiment, a gyroscope is used rather than a top. Its large rotating disc rotates with low friction about an axis which is fixed at a certain bearing point. A counterweight is adjusted in such a way that the bearing point coincides with the centre of gravity. If the gyroscope is in equilibrium and the disc is set spinning, the momentum L will be constant:

(1) $$L = I \cdot \omega_R$$

I: moment of inertia, ω_R: angular velocity

The moment of inertia of the rotating disc of the gyroscope is given by:

(2) $$I = \frac{1}{2} \cdot M \cdot R^2$$

M: mass of disc, R: radius of disc

If extra weight is put on the axis of rotation by addition of a mass m, the additional weight causes a torque τ which changes the angular momentum:

(3) $$\tau = m \cdot g \cdot r = \frac{\mathrm{d}L}{\mathrm{d}t}$$

r: distance from bearing point of axis of rotation to where the weight of the additional mass acts.

The axis of rotation then moves as shown in Fig. 2 by the following angle:

(4) $$\mathrm{d}\phi = \frac{dL}{L} = \frac{m \cdot g \cdot r \cdot dt}{L}$$

It also starts to precess. The angular velocity of the precession motion can then be derived:

(5) $$\omega_P = \frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{m \cdot g \cdot r}{L} = \frac{m \cdot g \cdot r}{I \cdot \omega_R}$$

where $\omega = 2\pi/T = 2\pi f$:

(6) $$\frac{1}{T_R} = f_R = \frac{m \cdot g \cdot r}{I} \cdot T_P$$

If the disc is set spinning in the absence of any extra external torque and the axis of rotation is slightly deflected to one side, the gyroscope will exhibit nutation. The angular velocity of the nutation is then directly proportional to the angular velocity of the rotation:

(7) $$\omega_N = C \cdot \omega_R \quad \text{and} \quad T_R = C \cdot T_N$$

C: constant

This experiment involves racing the rotational, precessive and nutative motions with the help of photoelectric light barriers, whereby the way the pulses change over time is recorded and displayed by 3B NET*log*™ and 3B NET*lab*™units.

EVALUATION

The periods of rotation, precession and nutation are determined from the recordings of how the pulses change over time. According to equation (6), the period of precession is inversely proportional to that of the rotation, while (7) says that the period of nutation is directly proportional to that of the rotation. On the respective graphs, the measured values will therefore lie along a straight line through the origin. From the slope of a line matched to these values $f_R(T_P)$ it is possible to obtain the moment of inertia of the gyroscope's rotating disc by experiment and then compare it with the theoretical value calculated using equation (2).

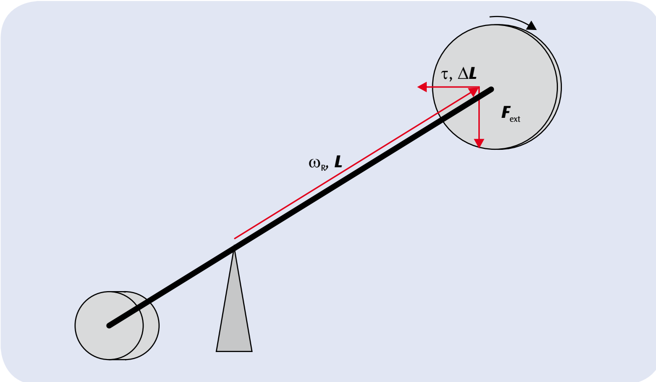

Fig. 1: Schematic of a gyroscope illustrating precession

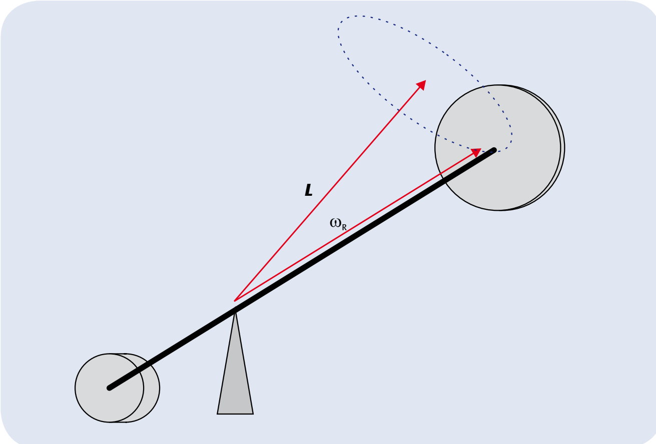

Fig. 2: Schematic of a gyroscope illustrating nutation

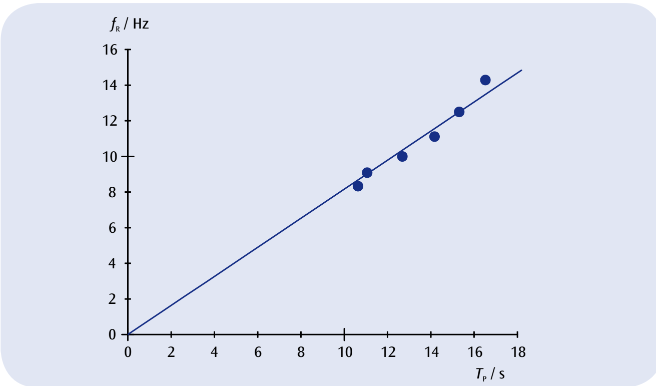

Fig. 3: Frequency of rotation f_R of a rotating disc as a function of the period of precession T_P.

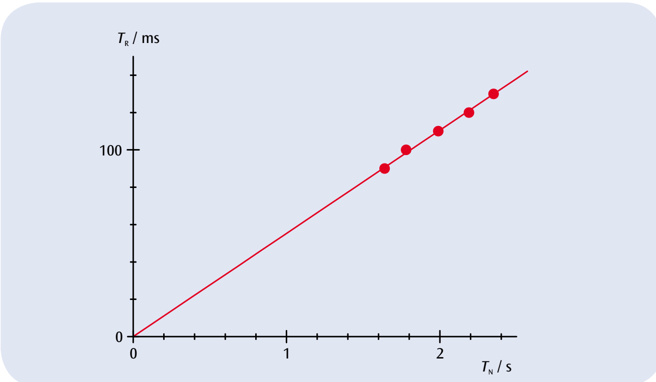

Fig. 4: Period of rotation T_R as a function of period of nutation T_N.

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