ADIABATIC INDEX OF AIR

OBJECTIVE

Determine the adiabatic index C_p / C_V for air using Rüchardt's method

SUMMARY

In this experiment an aluminium piston inside a precision-manufactured glass tube extending vertically from on top of a glass vessel undergoes simple harmonic motion on top of the cushion formed by the volume of air trapped inside the tube. From the period of oscillation of the piston, it is possible to calculate the adiabatic index.

EXPERIMENT PROCEDURE

- **Measure the period of oscillation of the aluminium piston.**
- **Determine the equilibrium pressure within the enclosed volume of air.**
- **Determine the adiabatic index of air and compare your result with the value quoted in literature.**

REQUIRED APPARATUS

Quantity	Description	Number
1	Mariotte Flask	1002894
1	Oscillation Tube	1002895
1	Mechanical Stopwatch, 15 min	1003369
1	Vacuum Hand Pump	1012856
Additionally recommended:		
1	Aneroid Barometer F	1010232
1	Callipers, 150 mm	1002601
1	Electronic Scale 200 g	1003433

BASIC PRINCIPLES

In a classic experiment designed by Rüchardt, it is possible to determine the adiabatic index for air from the vertical oscillations of a piston resting on a cushion of air inside a glass tube of constant cross-sectional area. The piston itself fits snugly and forms an air-tight seal. Disturbing the piston from its equilibrium position causes the air inside the tube to become expanded or compressed, causing the pressure inside to rise above or below atmospheric pressure, the effect of which is to restore the piston to its equilibrium position. The restoring force is proportional to the deviation from the equilibrium position, meaning that the piston exhibits simple harmonic oscillation.

Since there is no exchange of heat with the surroundings, the oscillations are associated with adiabatic changes of state. The following equation describes the relationship between the pressure p and the volume V of the enclosed air:

(1) $$p \cdot V^{\gamma} = \text{const.}$$

The adiabatic index γ is the ratio between the specific heat capacity at constant pressure C_p and constant volume C_V:

(2) $$\gamma = \frac{C_p}{C_V}$$

From equation (1), the following relationship can be derived for changes in pressure and volume Δp and ΔV

(3) $$\Delta p + \gamma \cdot \frac{p}{V} \cdot \Delta V = 0.$$

By substituting the internal crosssectional area A of the tube, the restoring force ΔF can be calculated from the change in pressure. Similarly the deflection of the piston from its equilibrium position can be determined from the change in volume.
Therefore, the following applies:

(4) $$\Delta F = -\gamma \cdot \frac{p}{V} \cdot A^2 \cdot \Delta s = 0.$$

This leads to the equation of motion for the oscillating piston.

(5) $$m \cdot \frac{d^2 \Delta s}{dt^2} + \gamma \cdot \frac{p}{V} \cdot A^2 \cdot \Delta s = 0$$

m = Mass of piston

Solutions to this classical equation of motion for simple harmonic oscillators are oscillations with the following period:

(6) $$T = 2\pi \sqrt{\frac{1}{\gamma} \cdot \frac{V}{p} \cdot \frac{m}{A^2}},$$

From this, the adiabatic index can be calculated as long as all the other variables are known.
In this experiment, a precision-made glass tube of small cross section A is set up vertically in a hole through the stopper for a glass vessel of large volume V and a matching aluminium piston of known mass m is allowed to slide up and down inside the tube. The aluminium piston exhibits simple harmonic motion atop the air cushion formed by the enclosed volume. It is possible to calculate the adiabatic index from the period of oscillation of the piston.

EVALUATION

The equilibrium volume V corresponds to the volume of the gas vessel, since that of the tube is small enough to be disregarded.

$$\gamma = \left(\frac{2\pi}{T}\right)^2 \cdot \frac{m}{A^2} \cdot \frac{V}{p}$$

The equilibrium pressure p is obtained from the external air pressure p_0 and the pressure exerted by the aluminium piston on the enclosed air in its rest state:

$$p = p_0 + \frac{m \cdot g}{A}, \text{ where } g = \text{acceleration due to gravity}$$

The expected result is therefore $\gamma = \frac{7}{5} = 1.4$, since air predominantly consists of diatomic molecules with 5 degrees of freedom for the absorption of heat energy.

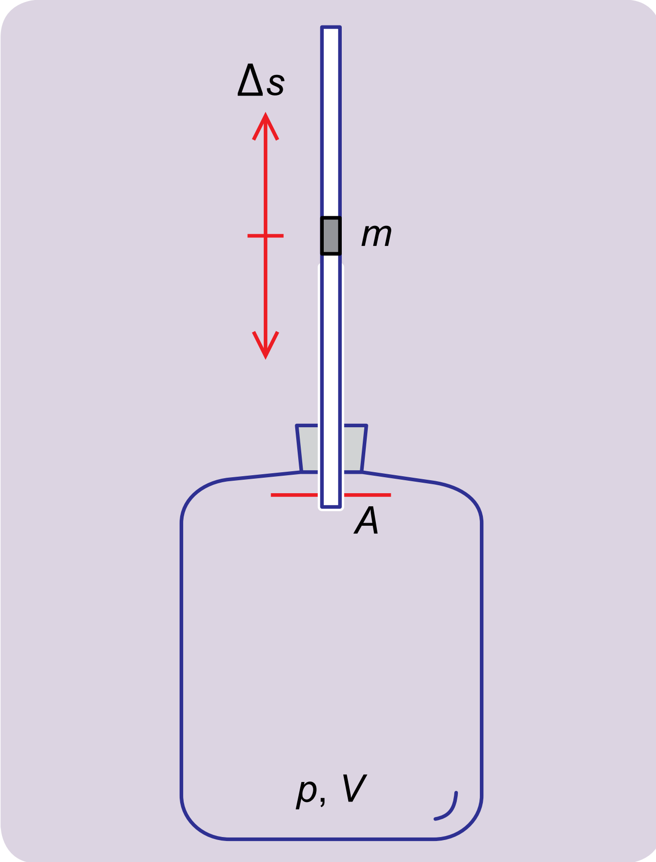

Fig. 1: Schematic of experiment set-up